

Exercise 89

Among all of the pairs of numbers whose difference is 12, find the pair with the smallest product. What is the product?

Solution

Let x and y be two real numbers difference is 12.

$$x - y = 12$$

Solve for y .

$$-y = -x + 12$$

$$y = x - 12$$

Now take the product of x and y , substitute the formula for y , and complete the square to write the quadratic function in vertex form.

$$\begin{aligned} P &= xy \\ &= x(x - 12) \\ &= x^2 - 12x \\ &= (x^2 - 12x + 36) - 36 \\ &= (x - 6)^2 - 36 \end{aligned}$$

Therefore, the smallest product is $P = -36$, which occurs when $x = 6$ and $y = 6 - 12 = -6$.